

# Unconditional quantum correlations do not violate Bell's inequality

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## Abstract

In this paper I demonstrate that the quantum correlations of polarization (or spin) observables used in Bell's argument against local realism have to be interpreted as *conditional* quantum correlations. By taking into account additional sources of randomness in Bell's type experiments, i.e., supplementary to source randomness, I calculate (in the standard quantum formalism) the complete quantum correlations. The main message of the quantum theory of measurement (due to von Neumann) is that complete correlations can be essentially smaller than the conditional ones. Additional sources of randomness diminish correlations. One can say another way around: transition from unconditional correlations to conditional can increase them essentially. This is true for both classical and quantum probability. The final remark is that classical conditional correlations do not satisfy Bell's inequality. Thus we met the following *conditional probability dilemma*: either to use the conditional quantum probabilities, as was done by Bell and others, or complete quantum correlations. However, in the first case the corresponding classical conditional correlations need not satisfy Bell's inequality and in the second case the complete quantum correlations satisfy Bell's inequality. Thus in neither case we have a problem of mismatching of classical and quantum correlations. It seems that the whole structure of Bell's argument was based on unacceptable identification of conditional quantum correlations with unconditional classical correlations.

## 1 Introduction

Bell's argument [1], [2] against the local realism played the great role in the quantum foundational Renaissance and tremendous development

of quantum technologies, especially in quantum information. One of the most attractive sides of this argument was its simplicity, logical, mathematical, and experimental (in the latter case, at least from the viewpoint of the experimental design). By analyzing the probabilistic structure of Bell's argument I immediately understood that, from the purely probabilistic viewpoint, the whole Bell story is about interrelation between conditional and unconditional probabilities, see, e.g., the first edition of my monograph [3]. The experimental probabilities of Bell's type have to be compared with *conditional classical probabilities* [4]. However, one cannot derive Bell's inequality for conditional (classical) probabilities. I tried to clarify this problem of interrelation between conditional and unconditional probabilities in a long series of works by using a variety of arguments, see, e.g., [5]–[9] and references herein. Now it becomes clear that the main problem was in concentration on the classical probabilistic counterpart of the problem. In particular, there was developed a very general contextual probability theory [5] in which Bell's inequality is violated, as well as other basic laws of classical (Kolmogorov [4], 1933) probability theory, e.g., the law of total probability. Violation of latter expresses interference in the purely probabilistic terms.

In this paper I proceed by applying solely *the standard quantum formalism of measurement theory*, due to von Neumann [10].

It will be shown that the quantum correlations of polarization (or spin) observables used in Bell's argument against local realism have to be interpreted as *conditional* quantum correlations. By taking into account additional sources of randomness in Bell's type experiments, i.e., supplementary to source randomness, cf. [8, 9] I calculate (in the standard quantum formalism) the complete quantum correlations. The main message of the quantum theory of measurement (due to von Neumann) is that the complete correlations can be essentially smaller than the conditional ones. Additional sources of randomness diminish correlations. One can say another way around: transition from unconditional correlations to conditional can increase them essentially. This is true for both classical and quantum probability. The final remark is that classical conditional correlations do not satisfy Bell's inequality. Thus we met the following *conditional probability dilemma*: either to use the conditional quantum probabilities, as was done by Bell [1], [2] and others, or the complete quantum correlations. However, in the first case the corresponding classical conditional correlations need not satisfy Bell's inequality and in the second case the complete quantum correlations satisfy Bell's inequality. Thus in neither case we confront the problem of mismatching of classical and quantum correlations. It seems that the whole structure of Bell's argument was based on

unacceptable identification of conditional quantum correlations with unconditional classical ones.

Therefore Bell's argument cannot be considered as an argument against local realism. From our viewpoint, the main message of quantum violation of Bell's inequality is encoded in the *Tsirelson bound*:

$$\text{UB}_{\text{QM}} = 2\sqrt{2}, \quad (1)$$

The classical probability theory cannot explain why conditional probabilities of some physical model violate Bell's inequality precisely up to this probabilistic constant  $\text{UB}_{\text{QM}}$ . The classical theory gives for conditional probabilities the upper bound

$$\text{UB}_{\text{CL}} = 4. \quad (2)$$

## 2 Interrelation of observations on a compound system and its subsystems

### 2.1 Averages

Consider a compound system  $\mathcal{S} = (S, S')$ , where  $S$  and  $S'$  have the state spaces  $H$  and  $K$ , respectively; thus  $\mathcal{S}$  is represented in the state space  $\mathcal{H} = H \otimes K$ . Let  $A_j, j = 1, \dots, k$ , be a group of observables on  $S$ , they are represented by Hermitian operators acting in  $H$  which are denoted by the same symbols. In general these observables are incompatible. Consider also an observable  $G$  on  $S'$ , having the values  $j = 1, \dots, k$ ; it is represented by a Hermitian operator in  $K$ ,  $G = \sum_j j P_j$ , where  $(P_j)$  is its spectral family consisting of mutually orthogonal projectors.

For any state  $\rho$ , a density operator in  $H$ , we can define the averages of observables  $A_j : M_j = \text{Tr} \rho A_j$ , and, for any state  $\sigma$ , a density operator in  $K$ , the averages of observables  $P_j : g_j = \text{Tr} \sigma P_j$ .

Now we consider the observables  $\mathcal{A}_j$  on the compound system  $\mathcal{S}$

$$\mathcal{A}_j = A_j \otimes P_j. \quad (3)$$

Consider a state  $R$  of  $\mathcal{S}$ . We can define the averages of the observables  $\mathcal{A}_j$  for this state:

$$m_j = \text{Tr} R \mathcal{A}_j. \quad (4)$$

Let the state of the compound system  $\mathcal{S}$  be factorisable, i.e., its subsystems are not entangled,

$$R = \rho \otimes \sigma. \quad (5)$$

Then

$$m_j = M_j g_j. \quad (6)$$

Suppose now that by experimenting with the compound system  $\mathcal{S}$  one “forgot” about the presence of the subsystem  $\mathcal{S}'$ .

*This forgetfulness has an interesting probabilistic effect: it induces the increase of averages, from  $m_j$  to  $M_j$  with the scaling coefficient  $k_k = 1/g_j$ .*

Thus if one treats an experiment on the compound system  $\mathcal{S}$  as an experiment on its proper subsystem  $\mathcal{S}$ , the averages and probabilities can increase essentially. For example, let  $\sigma = I/\dim K$ . Then  $g_j = \dim P_j/\dim K$  and  $k_j = \dim K/\dim P_j$ . We are, in fact, interested in the case  $\dim K = 4$  and  $\dim P_j = 1$ , i.e.,  $k_j = 4$ , the *four times increase of the magnitudes* of the averages and probabilities.

This increase of averages explains “the mystery of violations of Bell’s type inequalities and superstrong quantum correlations” (of course, only for a reader who is ready for my argument).

## 2.2 Correlations

Now move to the case of quantum correlations. Let now  $H = H_1 \otimes H_2$ , i.e.,  $\mathcal{S}$  is by itself a compound system  $\mathcal{S} = (S_1, S_2)$ , and let  $K = K_1 \otimes K_2$ , i.e.,  $\mathcal{S}'$  is by itself a compound system  $\mathcal{S}' = (S'_1, S'_2)$ . For the state space  $H_1$ , we consider a pair of observables  $A_0, A_1$  and, for the state space  $H_2$ , a pair of observables  $B_0, B_1$ ; for  $K_1$ , a pair of observables represented by orthogonal projectors  $P_0, P_1$  and, for  $K_2$ , a pair  $Q_0, Q_1$ . Finally, let  $\rho$  and  $\sigma$  be the states represented by density operators acting in  $H = H_1 \otimes H_2$  and  $K = K_1 \otimes K_2$ .

In Bell’s experimental scheme the observables in  $H_i$  represent polarization measurements and the observables in  $K_i$  represent measurements of outputs of random generators. The state  $\rho$  is the state of a pair of entangled photons  $\mathcal{S} = (S_1, S_2)$  and the state  $\sigma$  is a separable state of the pair of random generators, where the state of each random generator can (but need not) be given by a classical statistical mixture of two possible outputs. Of course, our scheme works for observables and random generators with an arbitrary number of outputs. By restricting the numbers of outputs to two we just try to keep closer to our concrete aim – Bell’s experimental scheme.

We introduce correlations in  $H$

$$C_{ij} = \text{Tr} A_i \otimes B_j \rho = \text{Tr} O_{ij} \rho, \quad (7)$$

where  $O_{ij} = A_i \otimes B_j$ , and in  $K$

$$g_{km} = \text{Tr} P_k \otimes Q_m \sigma = \text{Tr} O'_{km} \sigma, \quad (8)$$

where  $O'_{km} = P_k \otimes Q_m$ .

Now we consider the state space of the compound system  $\mathcal{S} = (S, S')$  given by  $\mathcal{H} = H \otimes K = H_1 \otimes H_2 \otimes K_1 \otimes K_2$ . For any state given by a density operator  $R$  on  $\mathcal{H}$ , we can find the correlation of the observables given by the operators  $O_{ij}$  and  $O'_{km}$ :

$$c_{ij} = \text{Tr} O_{ij} \otimes O'_{km} R = \text{Tr} A_i \otimes B_j \otimes P_k \otimes Q_m R. \quad (9)$$

Suppose that states in  $H$  and  $K$  are not entangled, i.e.,  $R = \rho \otimes \sigma$ . Then

$$c_{ij,km} = \text{Tr} O_{ij} \rho \text{Tr} O'_{km} \sigma = C_{ij} g_{km} \quad (10)$$

Suppose again that by experimenting with the compound system  $\mathcal{S}$  one “forgot” about the presence of the subsystem  $S'$ . If the correlation  $g_{km;\sigma} < 1$ , then:

*This forgetfulness induces the increase of correlations, from  $c_{ij,km}$  to  $C_{ij;\rho}$ .*

In this way one obtain “superstrong nonclassical Bell correlations”.

## 2.3 Towards proper quantum formalization of Bell’s experiment

Consider the case of the two dimensional spaces  $K_1$  and  $K_2$ . The corresponding bases in  $K_t, t = 1, 2$ , are  $(|0\rangle_t, |1\rangle_t)$ . To shorter notation, further we omit the index  $t$ . Let  $P_\alpha, Q_\alpha = |\alpha\rangle\langle\alpha|, \alpha = 0, 1$ . Let the states in  $K_1$  and  $K_2$  are neither entangled, i.e.,  $\sigma = \sigma_1 \otimes \sigma_2$  and each state  $\sigma_i$  is the “classical mixture”:

$$\sigma_1 = p_0|0\rangle\langle 0| + p_1|1\rangle\langle 1|, \quad \sigma_2 = q_0|0\rangle\langle 0| + q_1|1\rangle\langle 1|, \quad (11)$$

where  $p_0 + p_1 = 1$  and  $q_0 + q_1 = 1$  and nonnegative. Then

$$g_{km} = p_k q_m < 1.$$

In particular, if all probabilities are equal, we obtain that

$$g_{km} = 1/4. \quad (12)$$

This imply 4-times increase of correlations as the result of “missing” the subsystem  $S'$ .

In the Bell-type experiments, e.g., for the CHSH-inequality, one operates with the linear combination of correlations

$$C = C_{00} + C_{01} + C_{10} - C_{11} \quad (13)$$

It is convenient to represent  $C$  as the average of a single observable represented as the operator

$$\Gamma = O_{00} + O_{01} + O_{10} - O_{11} = A_0 \otimes B_0 + A_1 \otimes B_0 + A_0 \otimes B_1 - A_1 \otimes B_1. \quad (14)$$

Thus

$$C = \text{Tr} \rho \Gamma. \quad (15)$$

However, one can proceed with this operator only by ignoring the second subsystem of the compound system  $\mathcal{S} = (S, S')$ . By considering measurement on  $\mathcal{S}$  and by taking into account correlations of observables on  $S'$  we come to the representation of the corresponding modification of the correlation function  $C$  in the following operator form:

$$\gamma = O_{00} \otimes O'_{00} + O_{01} \otimes O'_{01} + O_{10} \otimes O'_{10} - O_{11} \otimes O'_{11}. \quad (16)$$

The corresponding correlation function is given by the average

$$c = \text{Tr} \rho \otimes \sigma \gamma = c_{00} + c_{01} + c_{10} - c_{11}, \quad (17)$$

where

$$c_{ij} = C_{ij} g_{ij} \quad (18)$$

In the case of equal probabilities  $p_i, q_j$ , see (12), we have:

$$c = C/4. \quad (19)$$

Thus by taking into account that the second system  $S'$  is also involved in the experiment we find that the “Bell correlation” function  $C$  decreases four times.

In particular, for the EPR-Bell correlations the Tsirelson bound for  $C$ , namely,  $C_{\max} = 2\sqrt{2}$ , which is the bound for conditional quantum correlations, leads to the following bound for unconditional quantum correlations

$$c = \frac{\sqrt{2}}{2} < 2. \quad (20)$$

### 3 Complete description of systems and observables involved in Bell’s type experiments

As was already emphasized, we want to account all physical systems and observables which are involved in Bell’s type experiments. Our main point is that in the standard quantum mechanical presentations,

e.g., in textbooks (but as well as in research papers) people forget about important physical systems playing the crucial role in the experiments. These are the random generators which outputs determine orientations of PBSs

We shall proceed with the CHSH inequality. For our purpose, it is useful to modify the modern experimental scheme. Typically one uses only two PBSs to realize four orientations, two at one side and two at another side. The needed two orientations for each of PBSs are produced with the help of two random generators  $G_1$  and  $G_2$ . Here  $G_1 = i, i = 0, 1$ , leads to the orientation  $i$  and hence the observation of  $A_i$  and  $G_2 = j, j = 0, 1$ , leads to the orientation  $j$  and hence the observation of  $B_j$ .

The crucial point is that *the outputs of random generators* also have to be considered as the results of measurements. Of course, generation of the pseudo-random numbers by a computer program which is often explored in Bell's type experiments dimmed the role of these observables. However, even in this case the values  $G_1 = i$  and  $G_2 = j$  have to be determined, so they can be treated as the results of the measurements of the reading type. If one uses physical devices giving random numbers, then the treatment of production of random numbers as measurements is straightforward.

This measurement viewpoint to outputs of random generators is better visible in the modified experimental design in which each setting is represented by its own PBS: the design with two PBSs at each side and each PBS is equipped with its own pair of detectors - in total 4 PBSs and 8 detectors combined with two stations  $C_1$  and  $C_2$  distributing signals from the source in accordance with the results of "random generators measurements".

Such an experimental design was, in particular, used by A. Aspect [11] (but with just one detector for each of four PBSs).

We present the corresponding citation of Aspect [12], see also [11], section "Difficulties of an ideal experiment":

"We have done a step towards such an ideal experiment by using the modified scheme shown on Figure 15. In that scheme, each (single-channel) polarizer is replaced by a setup involving a switching device followed by two polarizers in two different orientations:  $a$  and  $a'$  on side I,  $b$  and  $b'$  on side II. The optical switch  $C1$  is able to rapidly redirect the incident light either to the polarizer in orientation  $a$ , or to the polarizer in orientation  $a'$ . This setup is thus equivalent to a variable polarizer switched between the two orientations  $a$  and  $a'$ . A similar set up is implemented on the other side, and is equivalent to a polarizer switched between the two orientations  $b$  and  $b'$ . In our experiment, the distance  $L$  between the two switches was 13 m, and

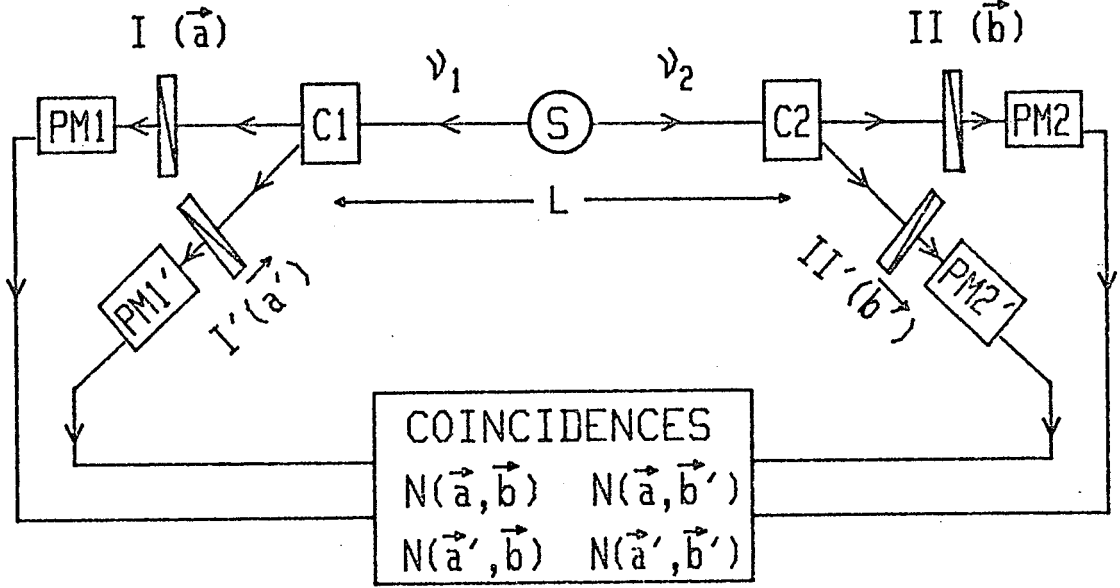


Figure 1: The scheme of the pioneer experiment of A. Aspect with four beam splitters [11].

$L/c$  has a value of 43 ns. The switching of the light was effected by home built devices, based on the acousto-optical interaction of the light with an ultrasonic standing wave in water. The incidence angle (Bragg angle) and the acoustic power, were adjusted for a complete switching between the 0th and 1st order of diffraction.”

So, there is no fundamental difference in proceedings with two or four PBSs. (Of course, we also can present our argument for experiments with only two PBSs, but here the measurement feature of random generation would be shadowed.)

### 3.1 Taking into account random choice of settings

As in [9], we consider the following experimental design:

- a). There is a source of entangled photons.
- b). There are 4 PBSs and corresponding pairs of detectors for each PBS, totally 8 detectors. PBSs are labeled as  $i = 1, 2$  (at the left-hand side, LHS) and  $j = 1, 2$  (at the right-hand side, RHS).
- c). Directly after source there are 2 distribution devices, one at LHS and one at RHS. At each instance of time,  $t = 0, \tau, 2\tau, \dots$  each



device opens the port to only one (of two) optical fibers going to the corresponding two PBSs. For simplicity, These switches are controlled by two random generators  $G_1$  (the left-hand side) and  $G_2$  (the right hand side ) with probabilities of for the  $i$ -channel,  $i = 0, 1$ , given by  $p_i$  and  $q_i$ , respectively ( $p_1 + p_2 = 1, q_1 + q_2 = 1$ ).

Now we introduce the physical observables measured in this experiment.

1)  $\mathcal{A}_i = \pm 1, i = 0, 1$  if the  $i$ -th channel (at LHS) is open and the corresponding (up or down) detector fires;

2)  $\mathcal{A}_i = 0$  if the  $i$ -th channel (at LHS) is blocked.

In the same way we define the “RHS-observables”  $\mathcal{B}_j = 0, \pm 1$ , corresponding to PBSs  $j = 1, 2$ .

Thus unification of 4 incompatible experiments of the CHSH-test into a single experiment modifies the range of values of polarization observables for each of 4 experiments; the new value, zero, is added to reflect the random choice of experimental settings. We emphasize that this value has no relation to the efficiency of detectors. In this model we assume that detectors have 100% efficiency. The observables take the value zero when the optical fibers going to the corresponding PBSs are blocked.

The measurement of the product of the observables  $\mathcal{A}_i \mathcal{B}_j$  is represented by the quantum operator  $O_{ij} \otimes O'_{ij}$ , where  $A_i$  and  $B_j$  are polarization observables corresponding to pairs of angles  $\theta_0, \theta_1$  and  $\theta'_0, \theta'_1$  and  $P_i, Q_j$  are one dimensional projectors which were defined in section 2.3. The state  $\rho$  is, for example, one of the Bell states.

Then the complete correlations are represented as  $c_{ij}$ , see (18), and the corresponding Bell correlation function  $c$ , see (20), does not exceeds the upper boundary 2. This is the upper boundary for unconditional classical correlations (obtained by Bell). Thus by taking into account randomness of selection of experimental settings we eliminate mismatching between classical and quantum correlations which was emphasized by Bell.

## 4 Quantum conditional correlations

Consider the same compound system  $\mathcal{S} = (S, S')$  as in section 2.2. We now remark that the joint measurement of the observables mathematically represented by the operators  $O_{ij} \otimes I$  and  $I \otimes O'_{km}$  can be treated as their sequential measurement. The correlations are the same. Now let us consider another problem – to find (again with the aid of the quantum theory of measurement) *conditional correlations*

of the observables  $A_i$  and  $B_j$  - conditioned to the fixed outputs of the measurements of the observables given by  $P_i$  and  $Q_j$ .

First we measure (on the subsystem  $S'$  of  $\mathcal{S}$ ) the observable  $O'_{ij} = P_i \otimes Q_j$ . In the quantum formalism this can be treated as measurement on  $\mathcal{S}$  of the observable given by  $I \otimes O'_{ij}$ . If the initial state of  $\mathcal{S}$  is  $R$ , then by getting the result ( $P_i = 1, Q_j = 1$ ) we know that the initial state is transformed to the post-measurement state:

$$R \rightarrow R_{ij} = \frac{(I \otimes O'_{ij})R(I \otimes O'_{ij})}{\text{Tr}(I \otimes O'_{ij})R(I \otimes O'_{ij})}. \quad (21)$$

Now in accordance with the quantum formalism for conditional measurements we measure the observable  $O_{ij} \otimes I$ . The corresponding (conditional) average is given by

$$C_{ij|\text{cond}} = \text{Tr} R_{ij} (O_{ij} \otimes I). \quad (22)$$

If  $R = \rho \otimes \sigma$ , then  $(I \otimes O'_{ij})R(I \otimes O'_{ij}) = \rho \otimes O'_{ij}\sigma O'_{ij}$ . In particular, the denominator in (21) is given by  $\text{Tr} O'_{ij}\sigma O'_{ij}$ . We also have:

$$C_{ij|\text{cond}} = \frac{\text{Tr} \rho O_{ij} \text{Tr} O'_{ij}\sigma O'_{ij}}{\text{Tr} O'_{ij}\sigma O'_{ij}} = \text{Tr} \rho O_{ij}. \quad (23)$$

Hence, the term  $\text{Tr} O'_{ij}\sigma O'_{ij}$  disturbing Bell's argument peacefully disappeared.

Thus, if one treats the correlations  $C_{ij}$  in Bell's correlation function  $C$ , see (14), as the quantum *conditional correlations*, then the diminishing effect discussed in section 2.3 disappears.

Now one might argue that in Bell's argument it is really possible to treat  $C_{ij}$  as conditional quantum correlations. However, there is a dangerous pitfall on this way of reasoning, namely, that Bell's inequality was proven for *unconditional classical correlation* [2], see also [5] for detailed analysis.

Is it possible to prove Bell's inequality for conditional classical correlations?

The answer is no. By using conditional correlations one can easily violate Bell's inequality, see [9] for details.

## 5 Concluding remarks

By using the standard quantum formalism we demonstrated that complete quantum correlations in Bell's type experiment do not violate Bell's inequality. It seems that Bell and following him scientists used

improper quantum mechanical description of such experiments. The conditional quantum correlations, where conditioning is to the choice of fixed experimental settings (e.g., the orientations of PBSs), were compared with unconditioned classical correlations.

From our analysis, it is clear that in Bell's framework there are two scientifically justified ways of proceeding:

- either with conditional quantum correlations (which are used in the literature on Bell's argument) and then compare them with classical conditional correlations,
- or with complete (unconditional) quantum correlations and then compare them with classical unconditional correlations.

In the first case, Bell's argument is collapsed, since classical conditional correlations can violate Bell's inequality; in the second case, it is collapsed, since complete (unconditional) quantum correlations satisfy Bell's inequality.

We remark that a few authors used (implicitly) classical conditioning (with respect to some parameters of the Bell-type experiments) to create classical models violating Bell's inequality, cf. with [13]–[20]. (This statement is not about the validity of concrete models, but about the essence of the method in the use.) I also think that conditioning on histories is the cornerstone of the interpretation of violation of Bell's inequality in the consistent histories approach [21].

From my viewpoint, the main message of Bell's considerations is encoded in the Tsirelson bound, see (1). The main problem is to find a physical explanation of the appearance this number in terms of classical conditional probability.

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